Studying the Full Solar Neutrino Parameter Space

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$\longrightarrow$ *IAS, Sept 2000*

A.F., hep-ph/0002063
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hep-ph/0002064, hep-ph/9910286

Outline

- Neutrinos from the Sun, standard analysis (brief summary)
  SSM predictions vs experimental rates, MSW and vacuum regions, day-night asymmetry; $(\sin^2 2\theta,\ \Delta m^2)$ *parameter space.*

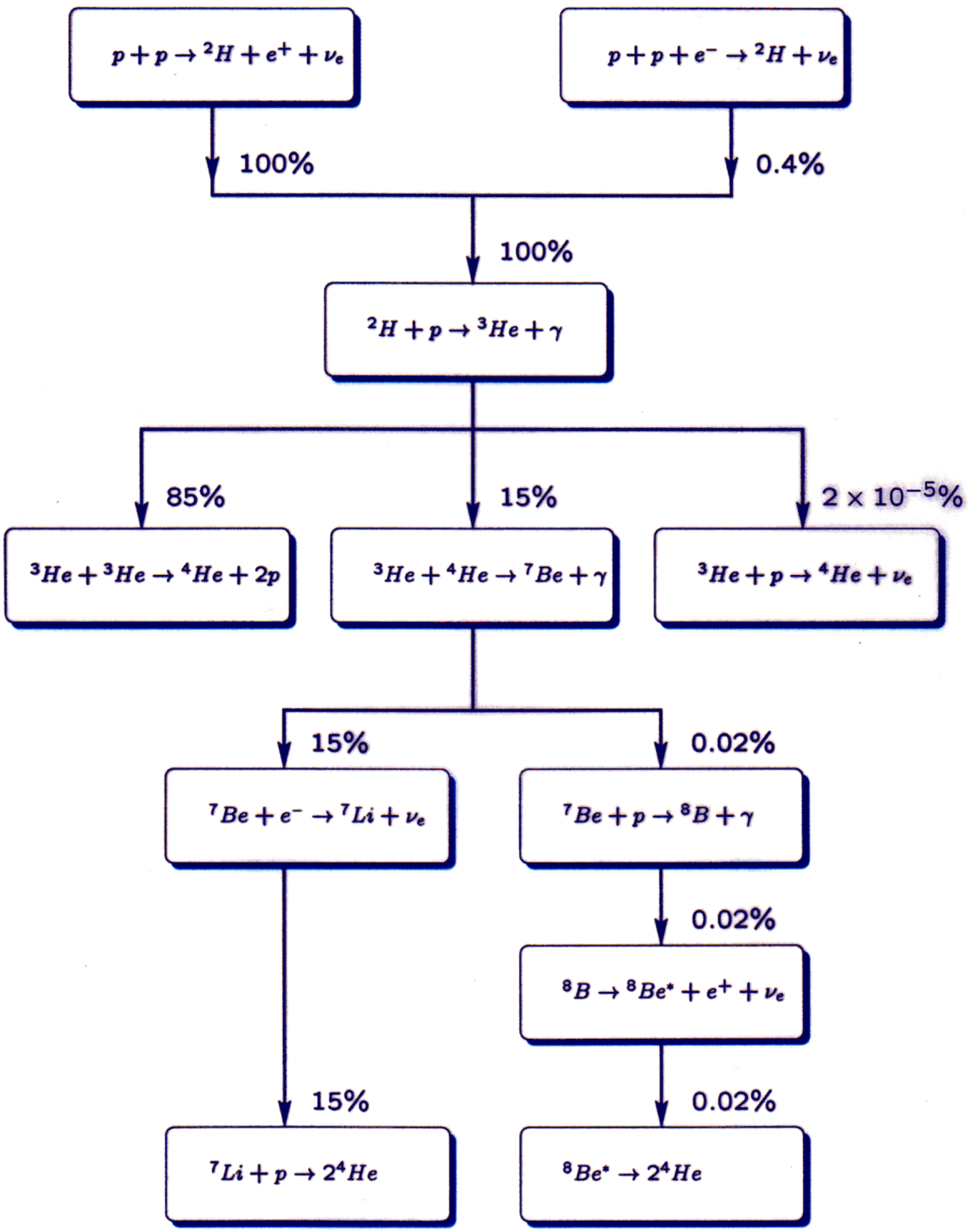
- The the full neutrino parameter space.
  $\pi/4 < \theta \leq \pi/2$, *different physics when matter effects are included. Why* $0 \leq \theta \leq \pi/2$, $\Delta m^2 > 0$ *more natural then* $0 \leq \theta \leq \pi/4$ *for both signs of* Δm^2.

- Solar matter effects are non-negligible for vacuum oscillations!

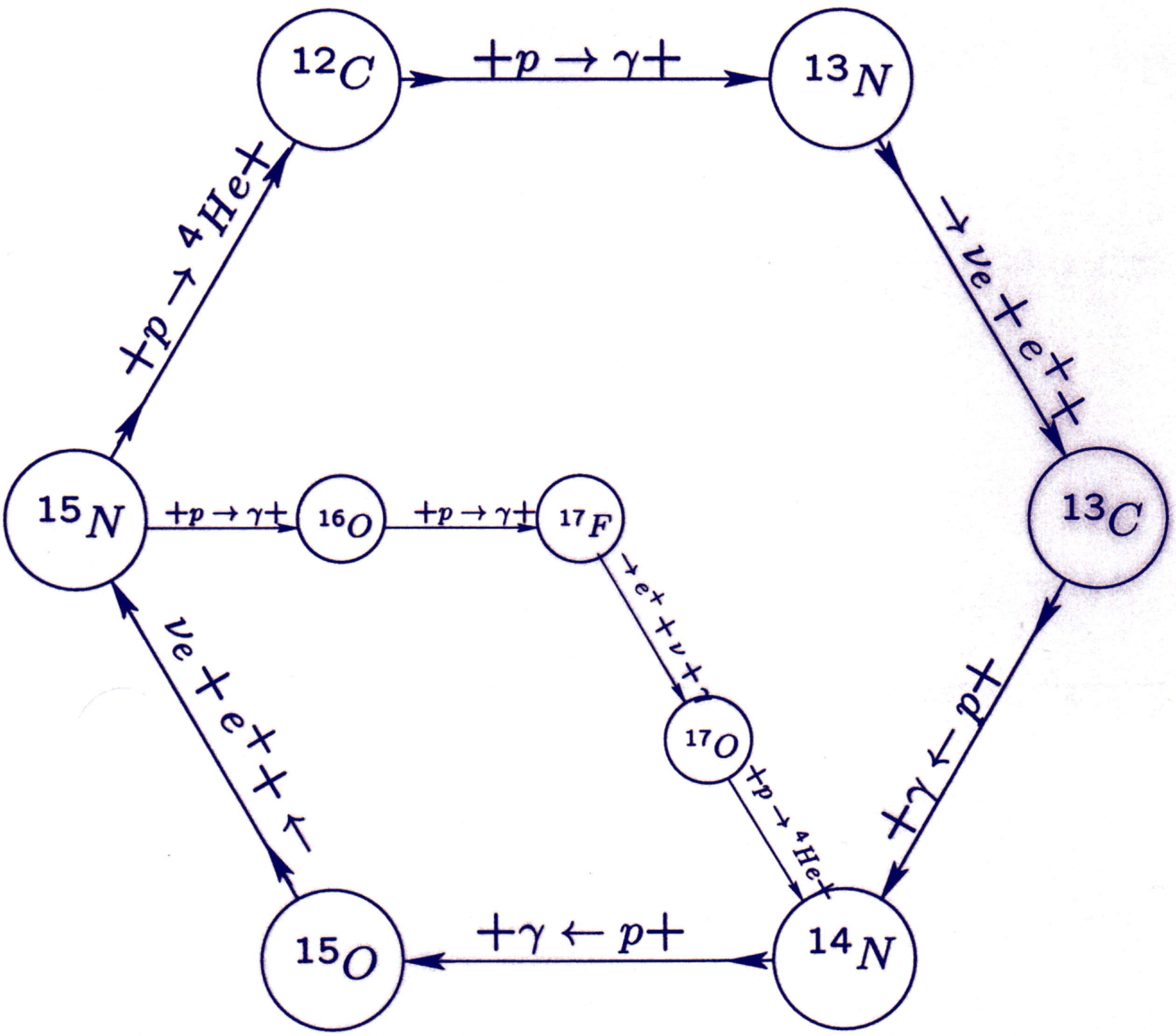
- MSW solutions are not necessarily confined to $\theta \leq \pi/4$.

- Global view of the 2–neutrino parameter space.

PP chain

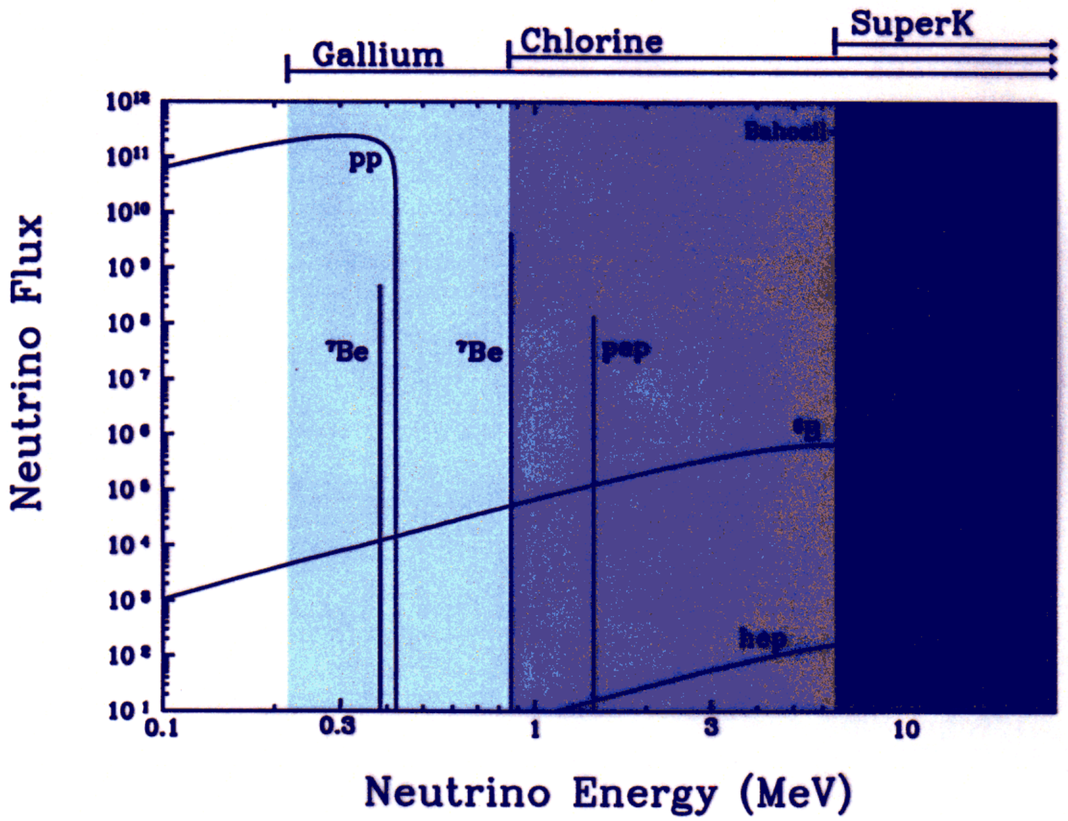

$p + p \to {}^2H + e^+ + \nu_e$
$p + p + e^- \to {}^2H + \nu_e$
100%
0.4%
100%
${}^2H + p \to {}^3He + \gamma$
85%
15%
$2 \times 10^{-5}\%$
${}^3He + {}^3He \to {}^4He + 2p$
${}^3He + {}^4He \to {}^7Be + \gamma$
${}^3He + p \to {}^4He + \nu_e$
15%
0.02%
${}^7Be + e^- \to {}^7Li + \nu_e$
${}^7Be + p \to {}^8B + \gamma$
0.02%
${}^8B \to {}^8Be^* + e^+ + \nu_e$
15%
0.02%
${}^7Li + p \to 2\,{}^4He$
${}^8Be^* \to 2\,{}^4He$

CNO cycle



^{12}C
$+p \rightarrow \gamma +$
^{13}N
$\rightarrow \nu e + e^+ +$
^{13}C
$+\gamma \leftarrow p+$
^{14}N
$+\gamma \leftarrow p+$
^{15}O
$\nu e + e^+ + \leftarrow$
^{15}N
$+p \rightarrow {}^4He+$
$+p \rightarrow \gamma +$
^{16}O
$+p \rightarrow \gamma +$
^{17}F
$\rightarrow e^+ + \nu +$
^{17}O
$+p \rightarrow {}^4He+$

Solar neutrino spectrum

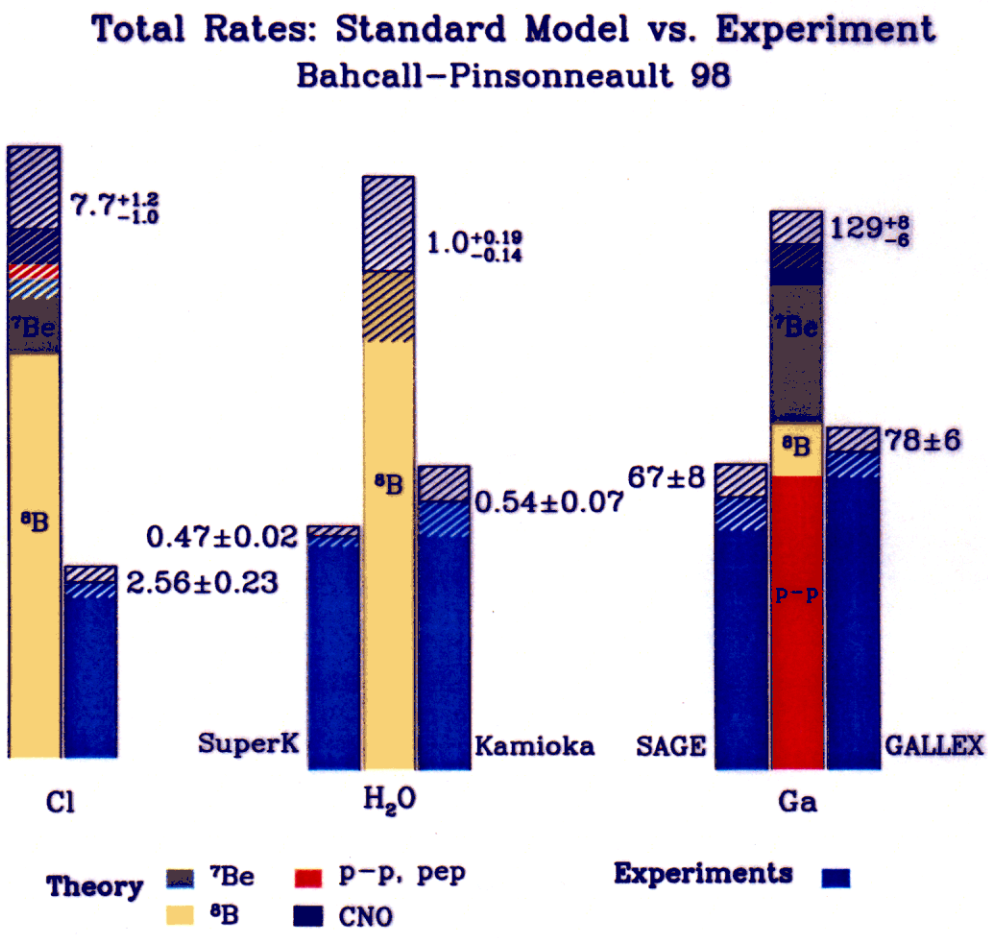


Solar neutrino energy spectrum

BP98 Standard Solar Model

http://www.sns.ias.edu/~jnb/

Comparison of theory and experiments



http://www.sns.ias.edu/~jnb/

The most attractive explanation of the observed deficit seems to be neutrino oscillations.

- The *weak eigenstates* are, in general, different from the *mass eigenstates*, just like in the quark sector:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \end{aligned}$$

- Evidence for neutrino oscillations has been observed in atmospheric neutrinos.

Two Mechanisms

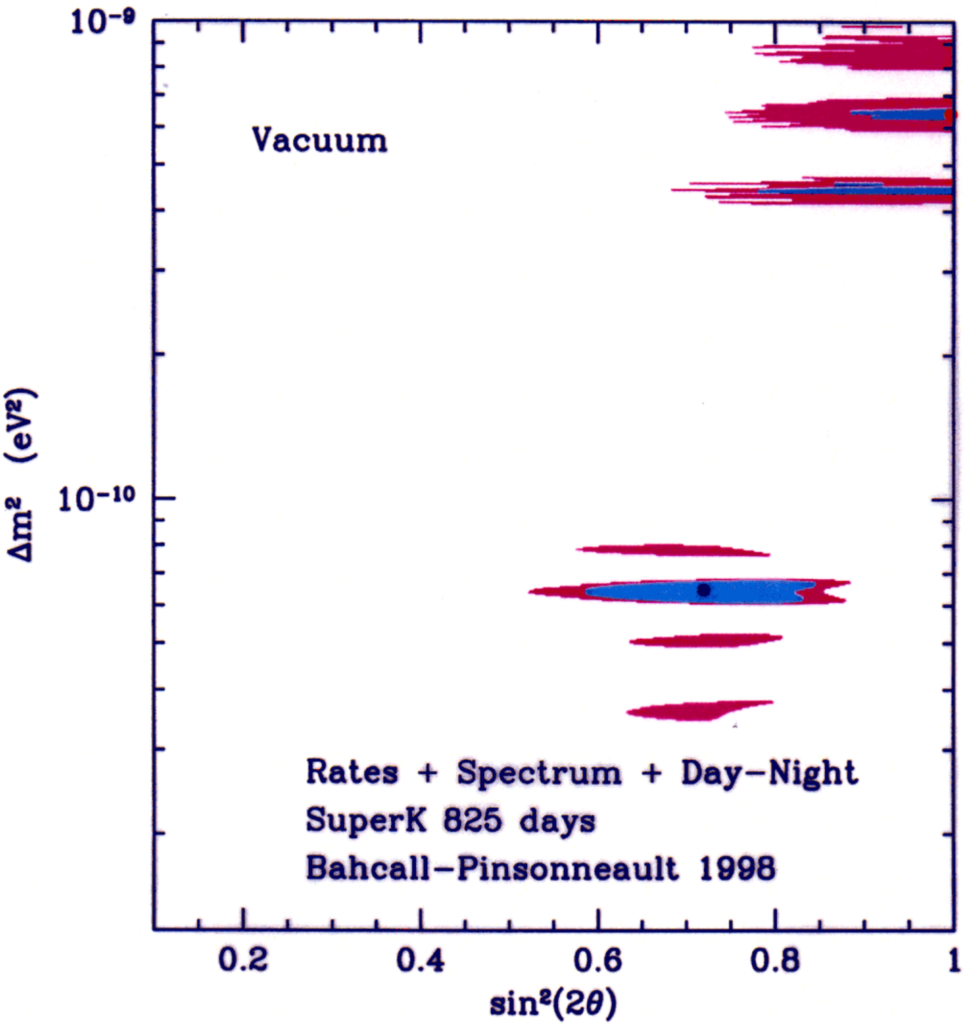
- Long-wavelength oscillations in vacuum

$$P = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2}{E}L\right)$$

  If oscillation length $\sim$ 1 astronomical unit, one can achieve the necessary energy-dependent pattern of suppression.

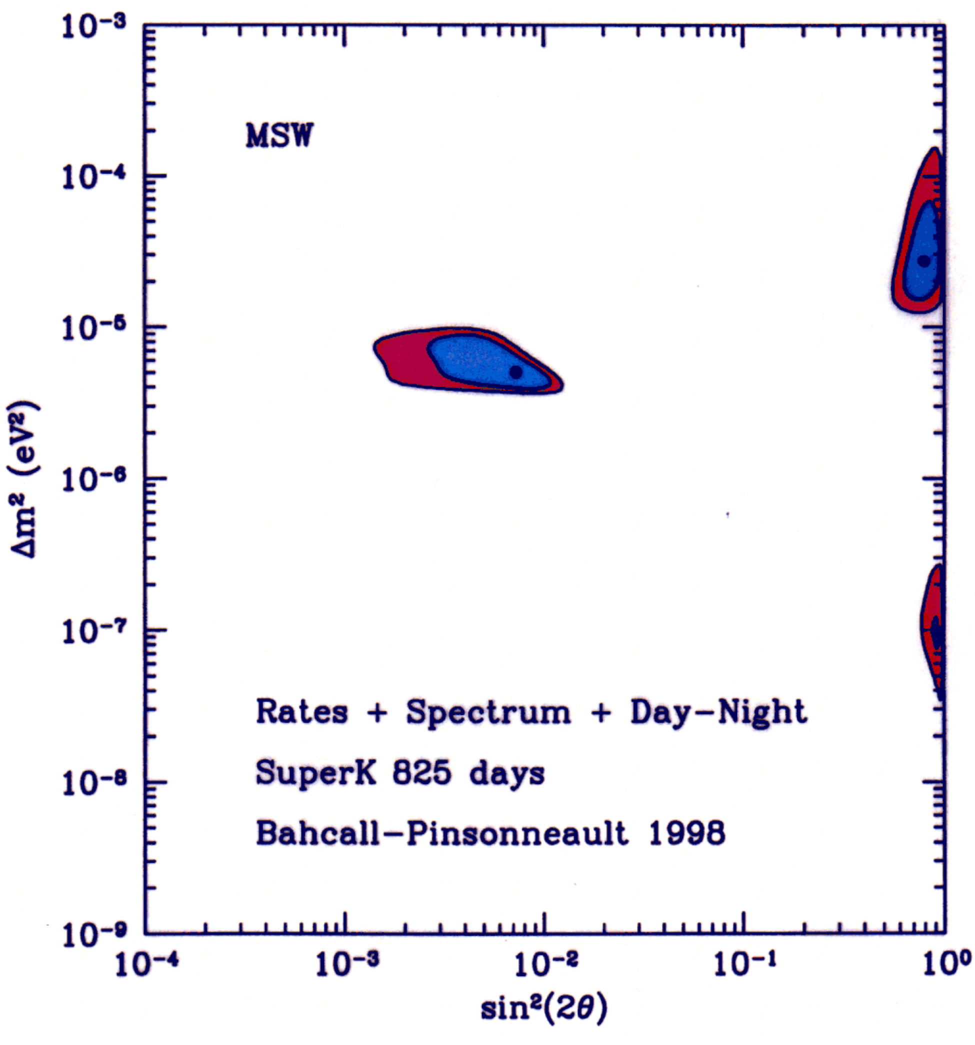
- Matter-enhanced flavor conversion in the Sun (+ matter effects in the Earth)

Vacuum oscillation solutions



John Bahcall, Plamen Krastev, Alexei Smirnov, *"WHAT WILL THE FIRST YEAR OF SNO SHOW?"*, hep-ph/9911248

MSW solutions



Day–night asymmetry

Among other well–studied phenomena is the effect of the Earth matter on the electron neutrino survival probability.

During the night solar neutrinos travel through the mantle and the outer core of the Earth. For a certain range of parameters the interactions with the Earth matter lead to regeneration of electron neutrinos. The resulting day–night asymmetry, if detected would be "smoking gun" signature of neutrino oscillations.

J. Bahcall, P. Krastev, *"Does the Sun Appear Brighter at Night in Neutrinos?"*, hep-ph/9706239

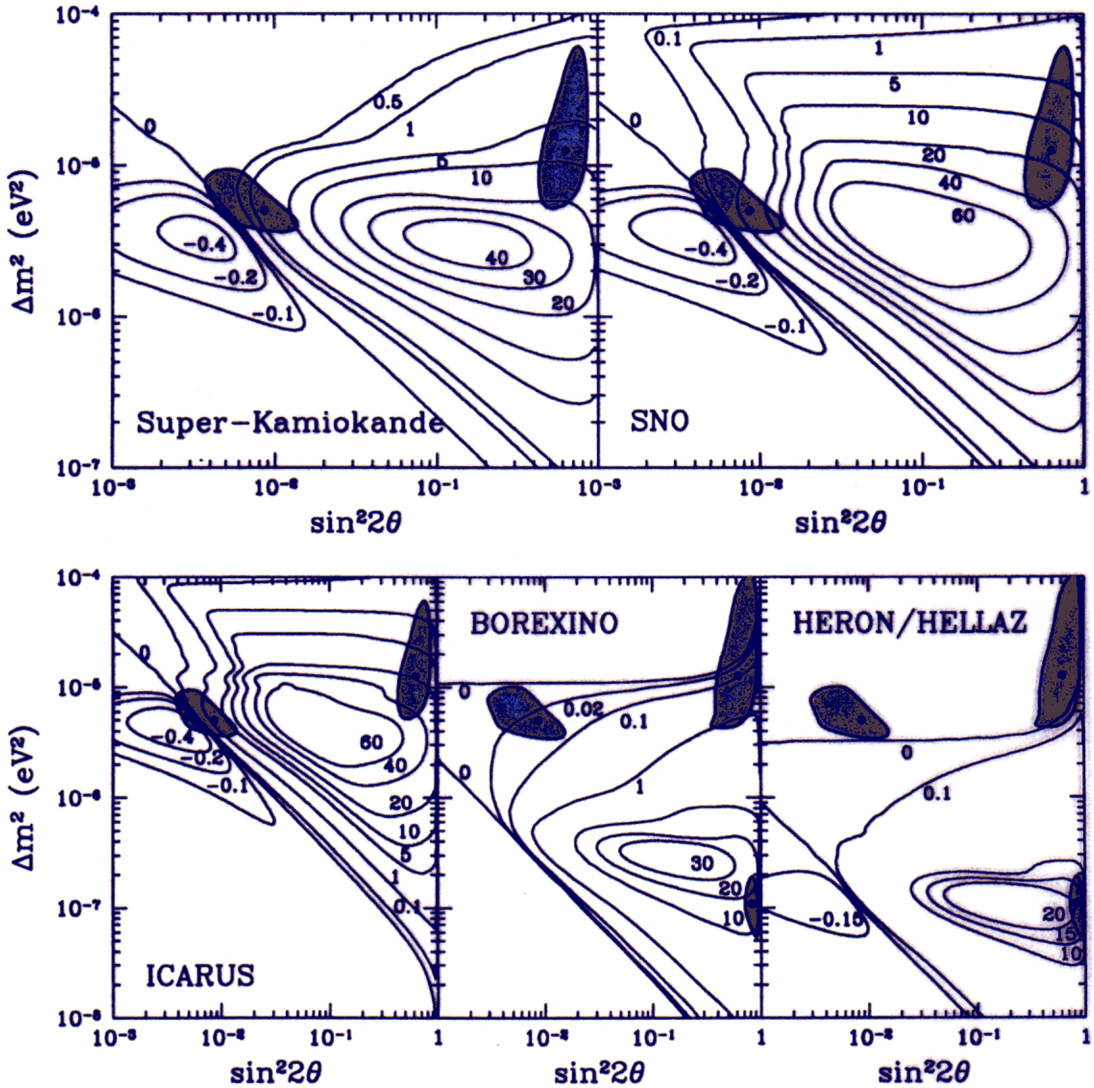


Figure 10

What happens to the day–night asymmetry at maximal mixing? In hep-ph/9706239 it was assumed that it vanishes. This was generally believed until the spring of 1999.

Alan H. Guth, Lisa Randall, Mario Serna, "DAY - NIGHT AND ENERGY VARIATIONS FOR MAXIMAL NEUTRINO MIXING ANGLES", hep-ph/9903464.

Pointed out that the asymmetry is nonzero at maximal mixing and continuous.

Question: Can one extend the scan beyond $\sin^2 2\theta = 1$, so all contours of D/N asymmetry would close?

Related: What is the physical range of θ?

Recall the definition

$$\begin{aligned}|\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle\end{aligned}$$

Invariant under $\theta \to \theta + \pi$, $|\nu_e\rangle \to -|\nu_e\rangle$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$, *i. e.* $[-\pi/2, \pi/2]$ and $[\pi/2, 3\pi/2]$ are equivalent.

Also invariant under $\theta \to -\theta$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$, $|\nu_2\rangle \to -|\nu_2\rangle$, hence $\theta \in [0, \pi/2]$.

Finally, it can also be made invariant under $\theta \to \pi/2 - \theta$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$ by relabeling the mass eigenstates $|\nu_1\rangle \leftrightarrow |\nu_2\rangle$, *i.e.* $\Delta m^2 \to -\Delta m^2$.

Thus,

- either $0 \leq \theta \leq \pi/2$ and $\Delta m^2 \geq 0$, or

- $0 \leq \theta \leq \pi/4$ and Δm^2 can have either sign.

What happens to the contours, if we extend the plot to $\theta > \pi/4$?

Can choose $\sin^2\theta$ as a variable and study $0 \leq \sin^2\theta \leq 1$.

Why is the $\theta > \pi/4$ half of the parameter space commonly neglected?

Possible reasons:

- For $\theta > \pi/4$ there is no level crossing in the Sun. The survival probability in the case of the MSW oscillations is always greater than 1/2.
  $\Longrightarrow$ *"not interesting"*

- The vacuum formula is symmetric with respect to $\theta \to \pi/2 - \theta$
  $\Longrightarrow$ *"the vacuum solutions in $\theta > \pi/4$ are mirror images"*

Next I'll reexamine both of these points.

Hamiltonian in matter

For electron neutrino oscillations into another active species, the Hamiltonian has the form

$$\text{const} + \begin{pmatrix} -\frac{\Delta m^2}{4E_\nu}\cos 2\theta + \sqrt{2}G_F N_e(x) & \frac{\Delta m^2}{4E_\nu}\sin 2\theta \\ \frac{\Delta m^2}{4E_\nu}\sin 2\theta & \frac{\Delta m^2}{4E_\nu}\cos 2\theta \end{pmatrix}.$$

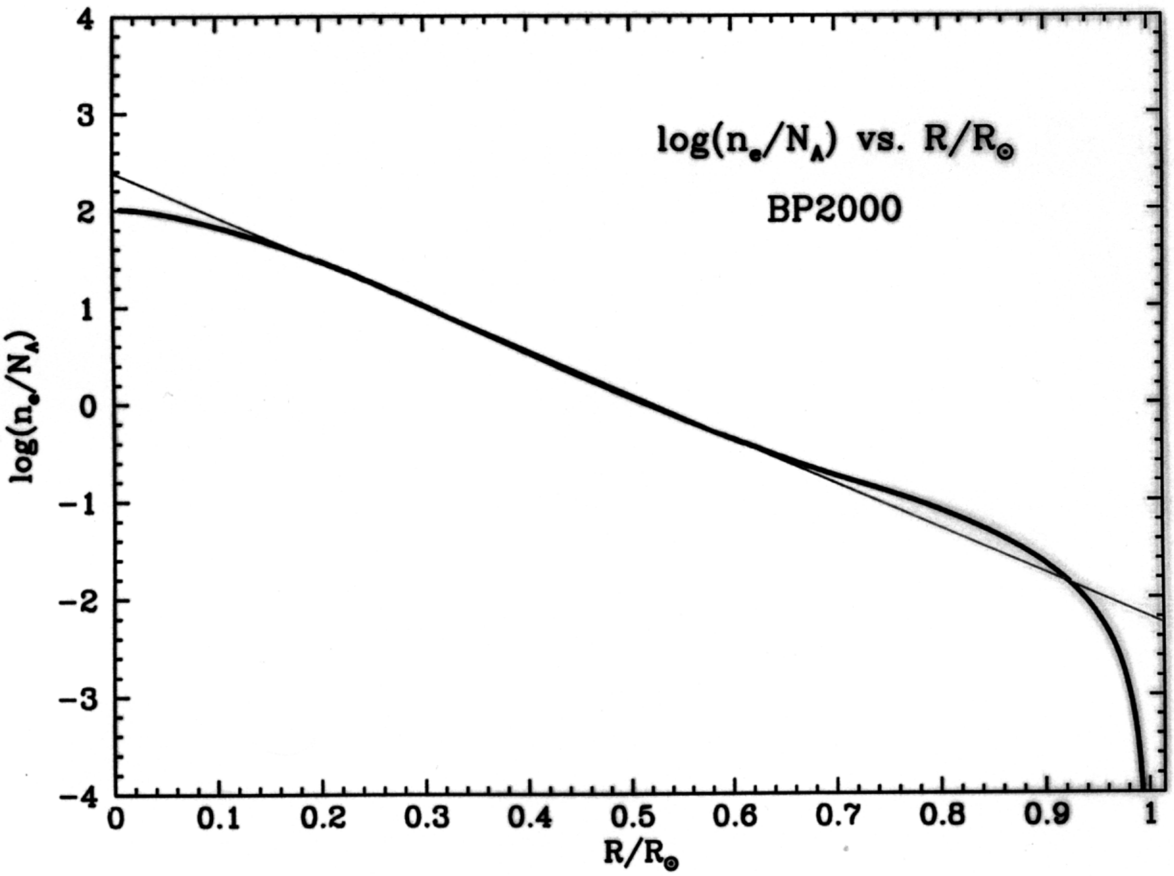
The eigenvalues of the new matrix are

$$\frac{\Delta\tilde{m}^2}{4E_\nu} = \pm\left[\left(\frac{\Delta m^2}{4E_\nu}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E_\nu}\cos 2\theta - \frac{\sqrt{2}G_F N_e(x)}{2}\right)^2\right]^{1/2},$$

and the new mixing angle is given by

$$\cos 2\theta_M(x) = \frac{\frac{\Delta m^2}{4E_\nu}\cos 2\theta - \frac{\sqrt{2}G_F N_e(x)}{2}}{\sqrt{\left(\frac{\Delta m^2}{4E_\nu}\right)^2 \sin^2 2\theta + \left(\frac{\Delta m^2}{4E_\nu}\cos 2\theta - \frac{\sqrt{2}G_F N_e(x)}{2}\right)^2}}$$

Electron density profile of the Sun



BP2000 Standard Solar Model

Level jumping probability

The level jumping probability P_c depends on the rate of change of the electron density in the Sun.
In the exponential profile approximation ($n_e \propto \exp(-r/r_0)$) it is given by

$$P_c = \frac{e^{\gamma \cos^2\theta} - 1}{e^{\gamma} - 1},$$

$$\gamma = 2\pi r_0 \frac{\Delta m^2}{2E_\nu} = 1.22 \left(\frac{\Delta m^2}{10^{-9}\mathrm{eV}^2}\right)\left(\frac{0.863\mathrm{MeV}}{E_\nu}\right).$$

($r_0 = R_\odot/10.54$)

- *adiabatic:* $P_c \to 0$; true when $\Delta m^2 \gg 10^{-9}$ eV$^2(E_\nu/1$ MeV), $\sin^2\theta \gg (10^{-9}$ eV$^2/\Delta m^2)$ $(E_\nu/1$ MeV).

- *extreme nonadiabatic:* $P_c \to \cos^2\theta$; true in the opposite limit.

The survival probability for neutrinos arriving at the Earth (for $\Delta m^2 \lesssim 10^{-5}$ eV^2)

$$\begin{aligned} P &= P_c \cos^2\theta + (1 - P_c)\sin^2\theta \\ &+ \sqrt{P_c(1-P_c)}\sin 2\theta \cos\left(2.54\frac{\Delta m^2 L}{E_\nu} + \delta\right), \end{aligned}$$

The last term is responsible for oscillations in vacuum. For sufficiently large mass splitting ($\Delta m^2 \gtrsim 6 \times 10^{-9}$ eV^2) this term averages out to zero upon integration over energy.

Limits:

- *adiabatic:* $P_c \to 0$, $P \to \sin^2\theta$.

- *extreme nonadiabatic:* $P_c \to \cos^2\theta$,

$$P \to 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2}{E}L\right).$$

For $\Delta m^2/E \gtrsim 10^{-5}$ eV^2/MeV one should take into account that $\theta_\odot < \pi/2$ in the core. Then

$$P = P_1 \cos^2\theta + (1 - P_1)\sin^2\theta,$$

where $P_1 = P_c \sin^2\theta_\odot + (1 - P_c)\cos^2\theta_\odot$.

For this Δm^2 $\gamma >> 1$, *i.e.* $P_c = 0$ and

$$P = \cos^2\theta_\odot \cos^2\theta + \sin^2\theta_\odot \sin^2\theta.$$

P increases with Δm^2.

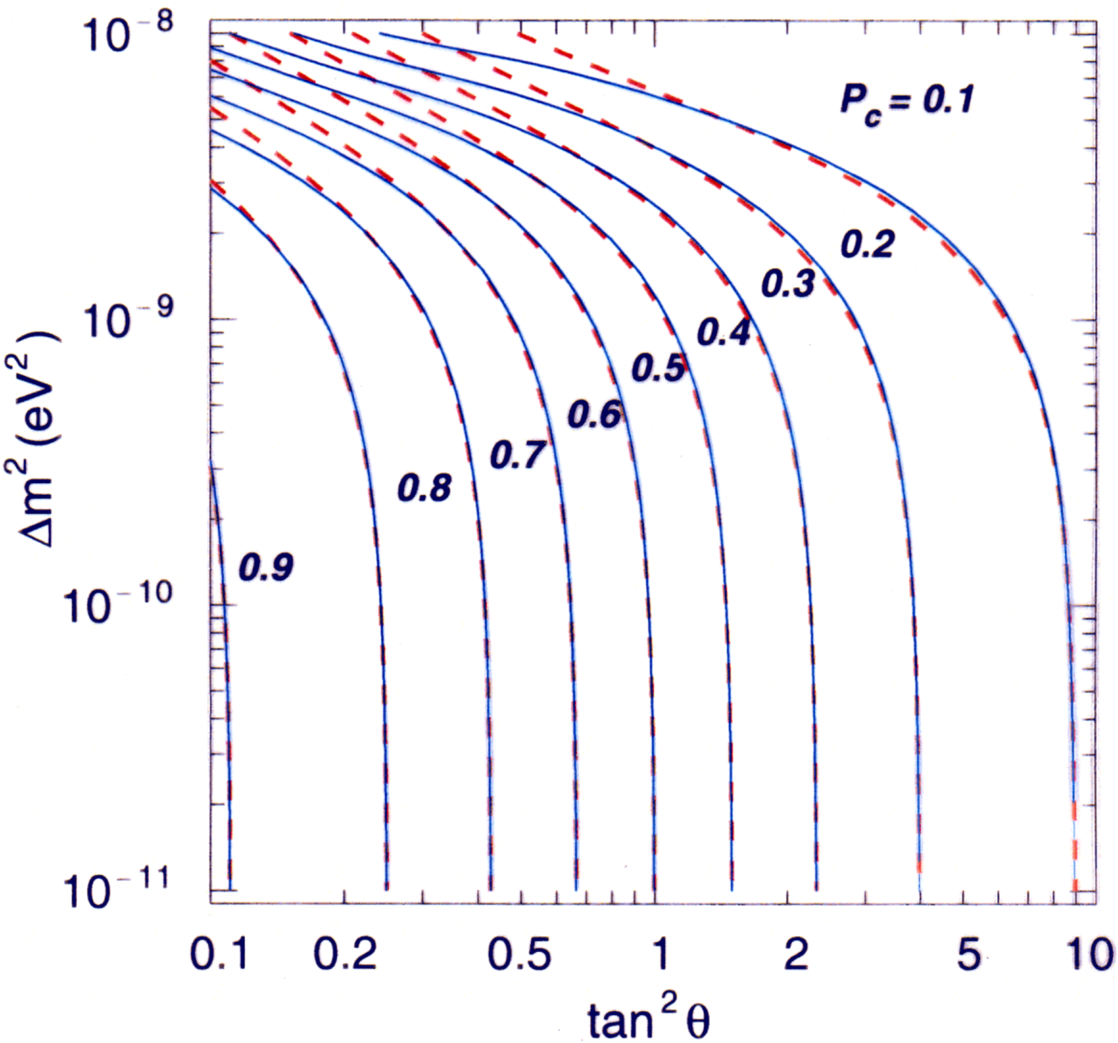
In the cases of vacuum oscillation solution, Δm^2 is low and the extreme non-adiabatic limit is reached.

Or is it?

For low Δm^2 the relevant region is close to the Sun's surface, where the exponential approximation is bad.

The most reliable way is to compute P_c numerically.

Level jumping probability in the vacuum region for $E_\nu = 0.863$ MeV (^{7}Be neutrino).



Notice that the variable is $\tan^2\theta$! Symmetric under $\theta \to \pi/2 - \theta$ *on the log scale.*

Why choose $\tan^2\theta$?

- We saw that $\sin^2 2\theta$ is not very good
- We would like points θ and $\pi/2-\theta$ to be symmetric
- We also would like to use *a log scale*, to fit all solutions on the same plot

$\tan^2\theta$ is just such a variable.

Analytical expression for the correction

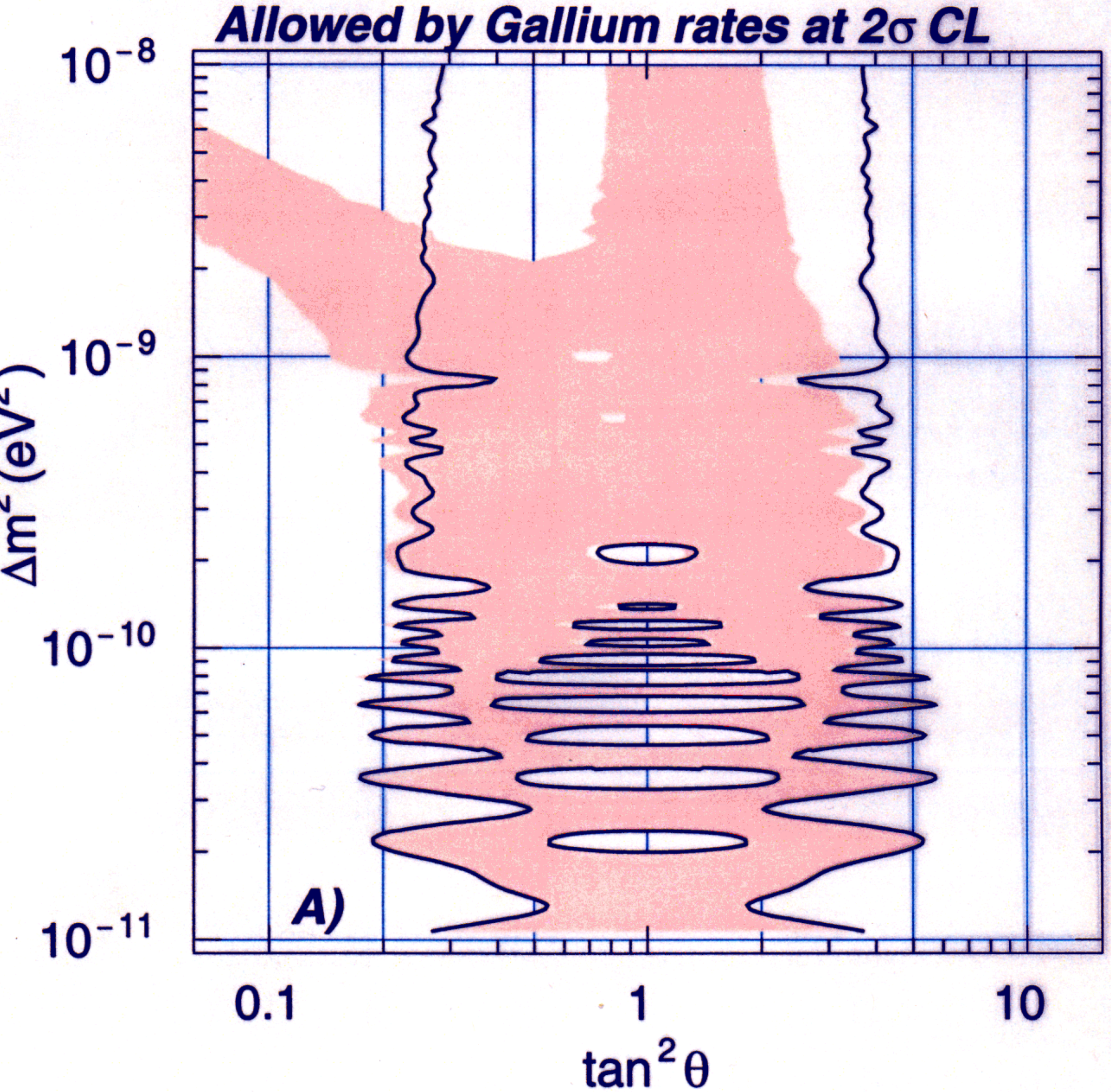
Retaining in the formula for P_c also terms linear in γ, we obtain

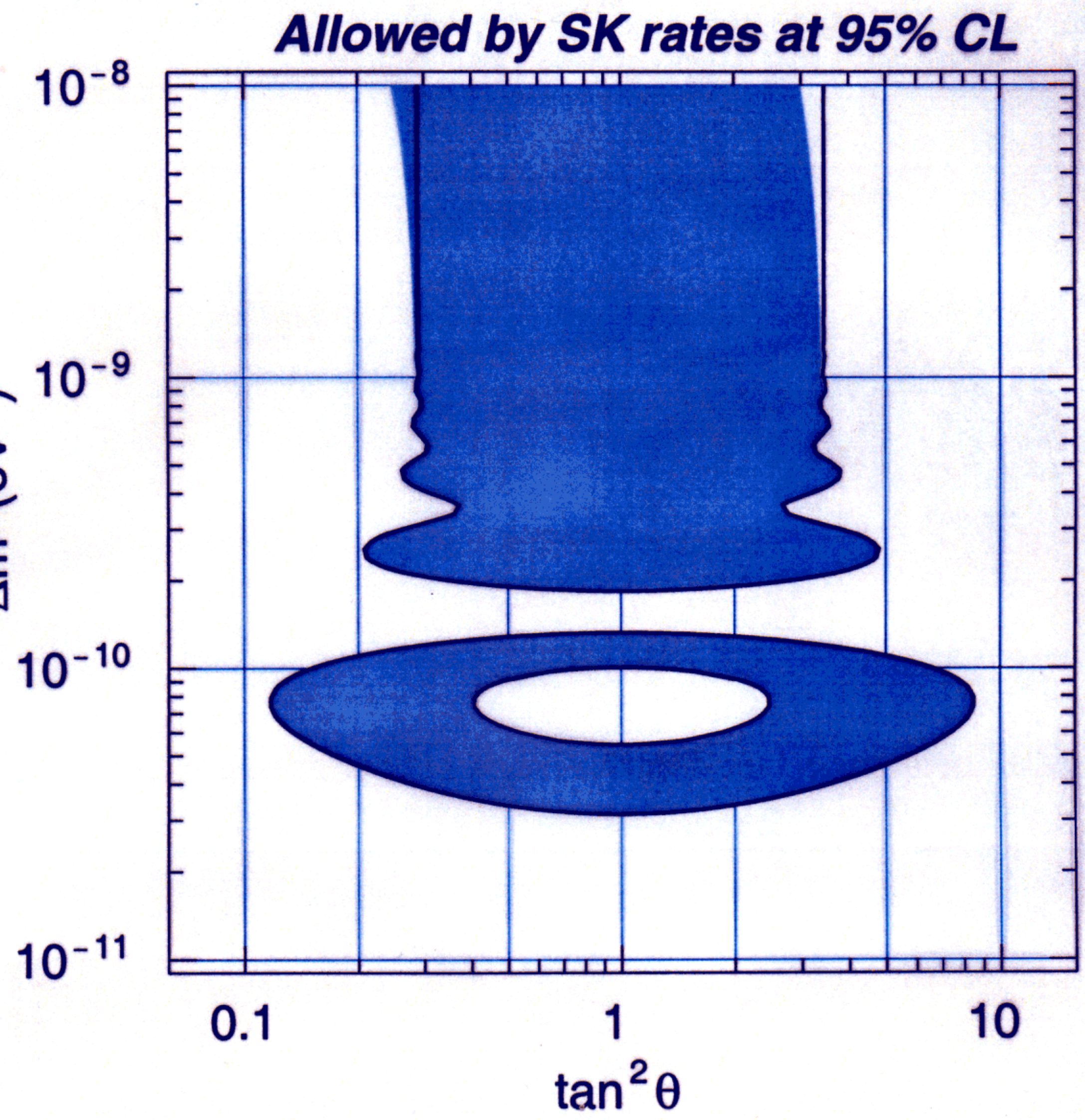
$$\begin{aligned} P &= 1 - \left(1 + \frac{\gamma}{4}\cos 2\theta\right) \times \\ &\times \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E} + \delta\right) \\ &+ O(\gamma^2) \end{aligned}$$

The term linear in γ contains $\cos 2\theta$, which is manifestly non-invariant under $\theta \to \pi/2 - \theta$.

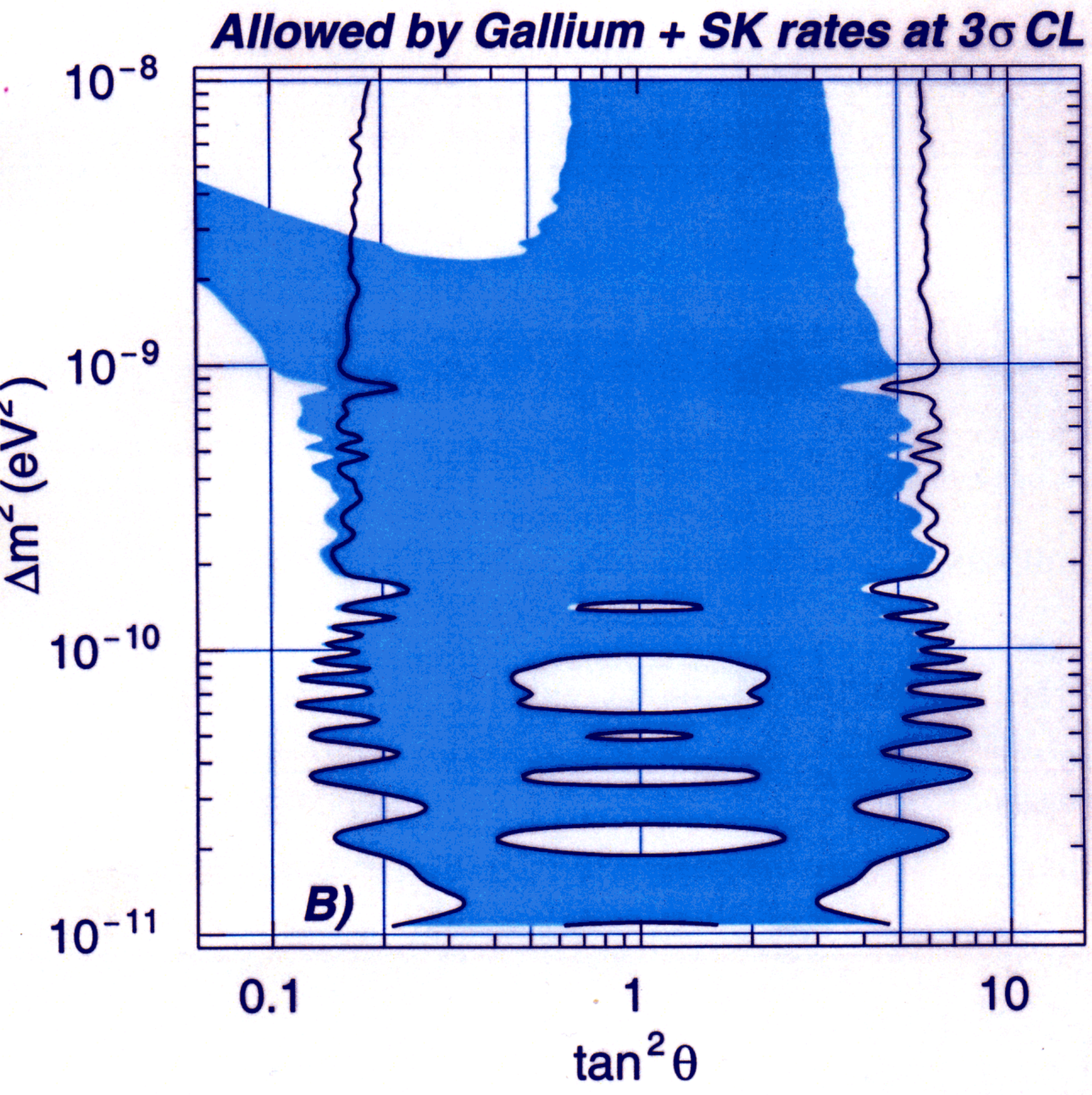
For very small Δm^2 the formula for the exp. profile can be used with $r_0 = R_\odot/18.4$

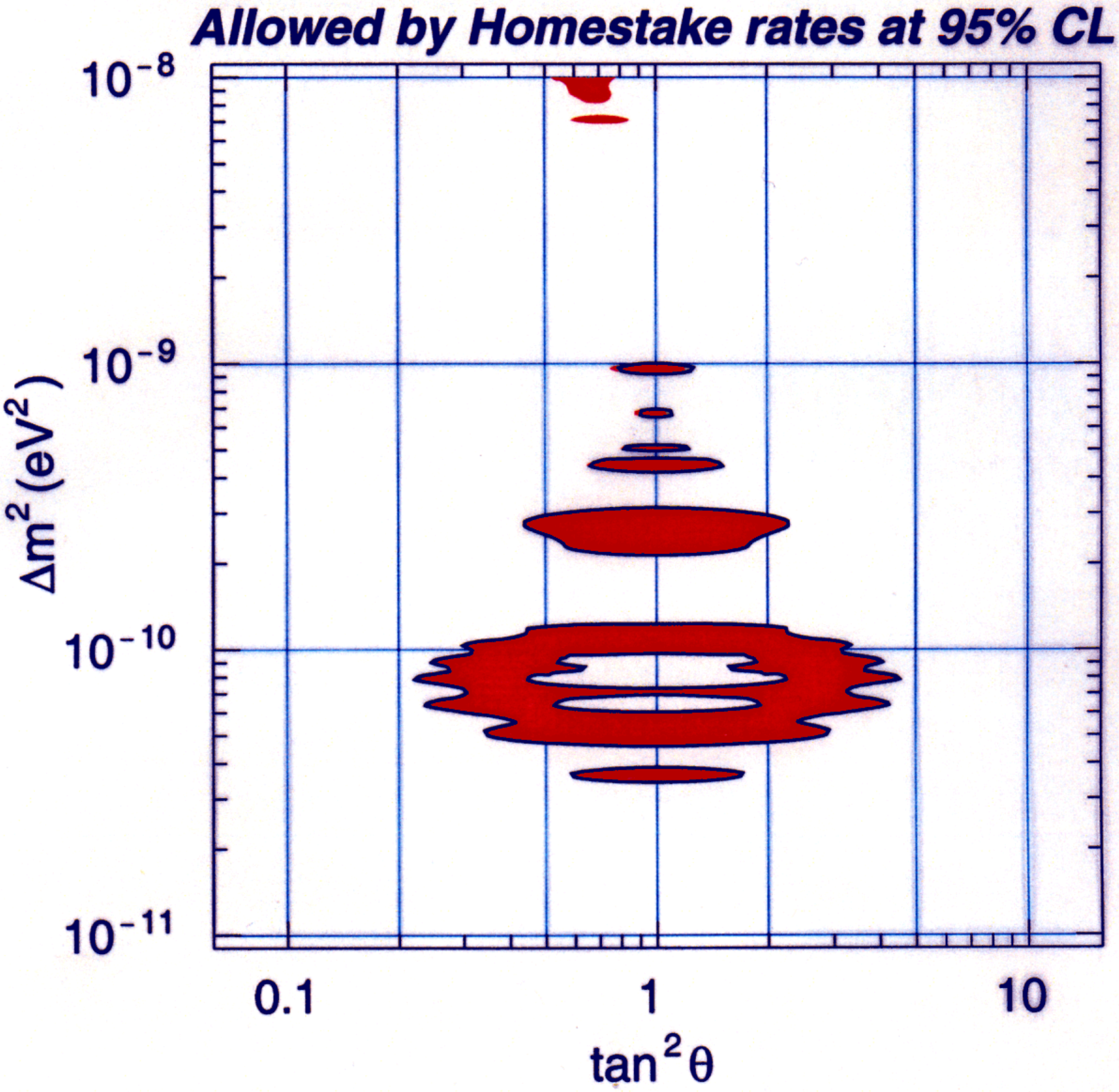
Notice, that trying to guess the value of r_0 by measuring the slope of the density profile at the resonance point gives an inaccurate result.

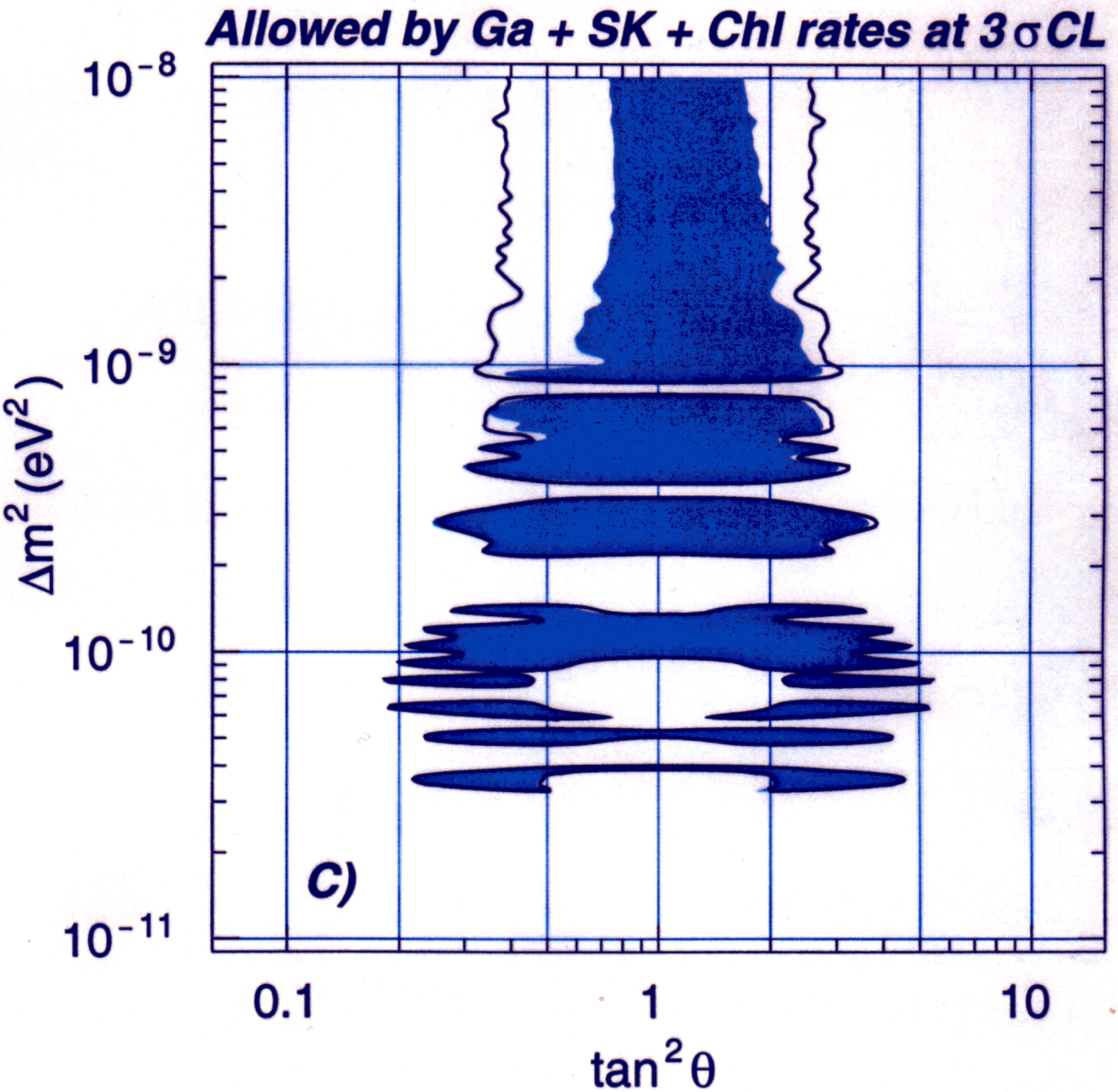

Allowed by Gallium rates at 2σ CL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Δm^2 (eV^2)
A)
0.1
1
10
$\tan^2\theta$



Allowed by SK rates at 95% CL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\Delta m^2 (eV^2)$
0.1
1
10
$\tan^2\theta$

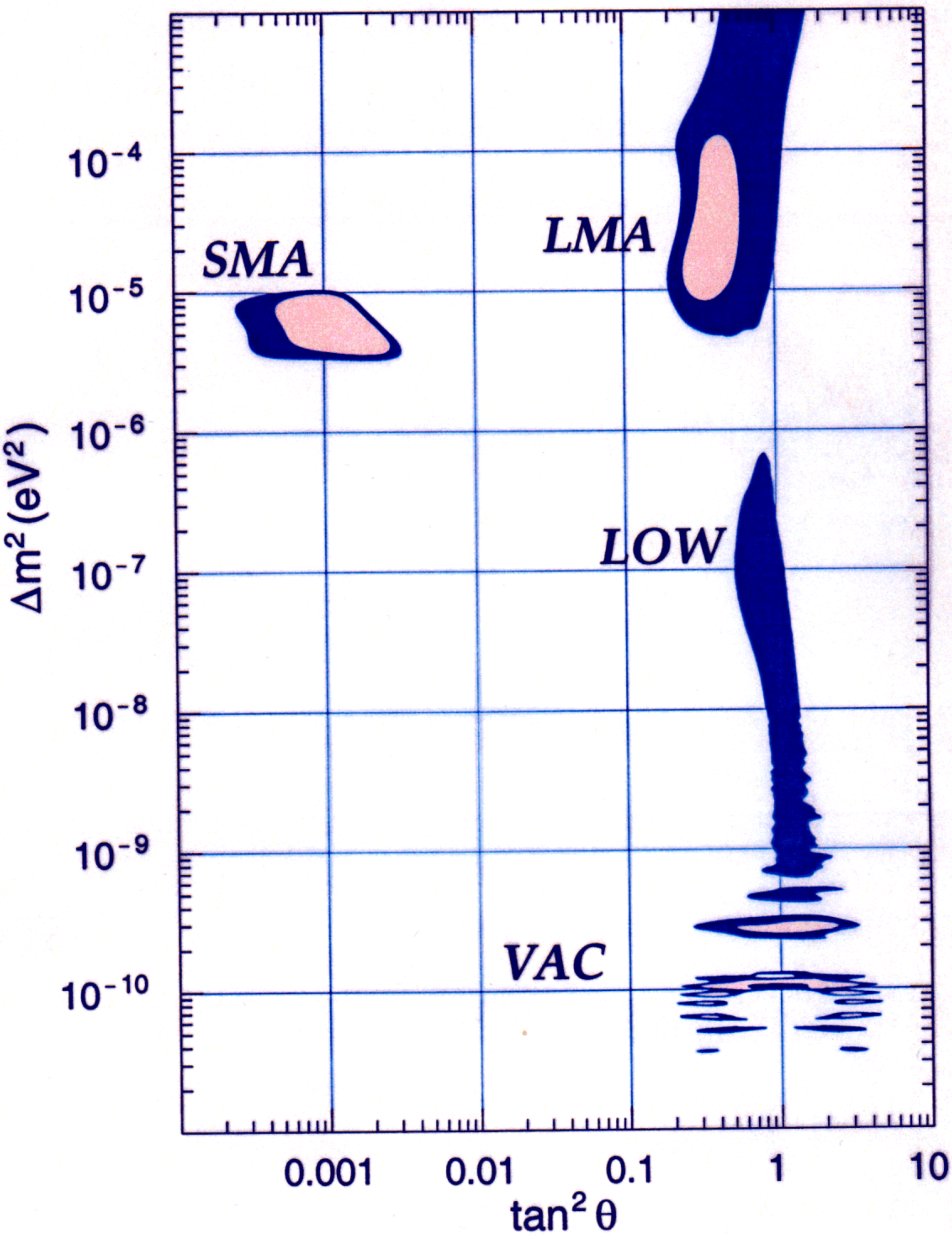

Allowed by Gallium + SK rates at 3σ CL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Δm^2 (eV^2)
B)
0.1
1
10
$\tan^2\theta$


Allowed by Homestake rates at 95% CL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Δm^2 (eV^2)
0.1
1
10
$\tan^2\theta$

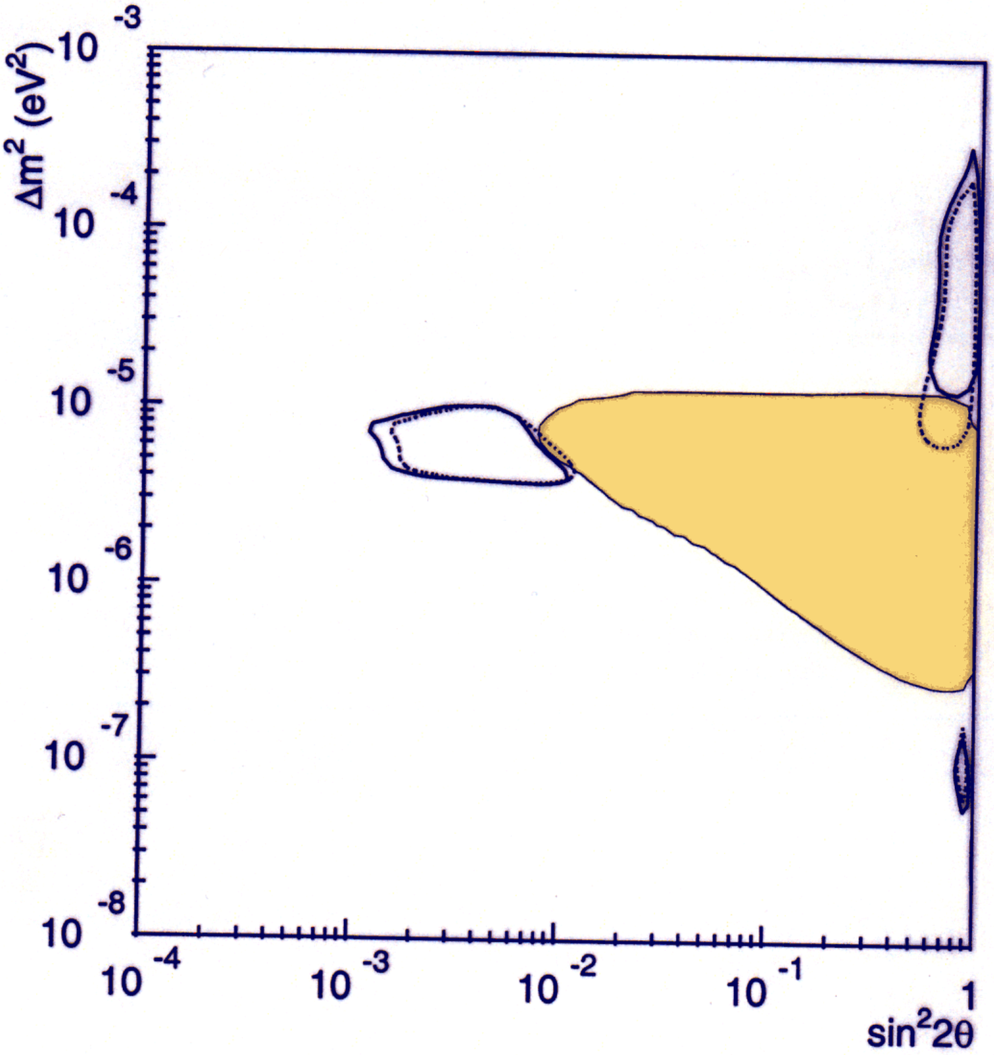


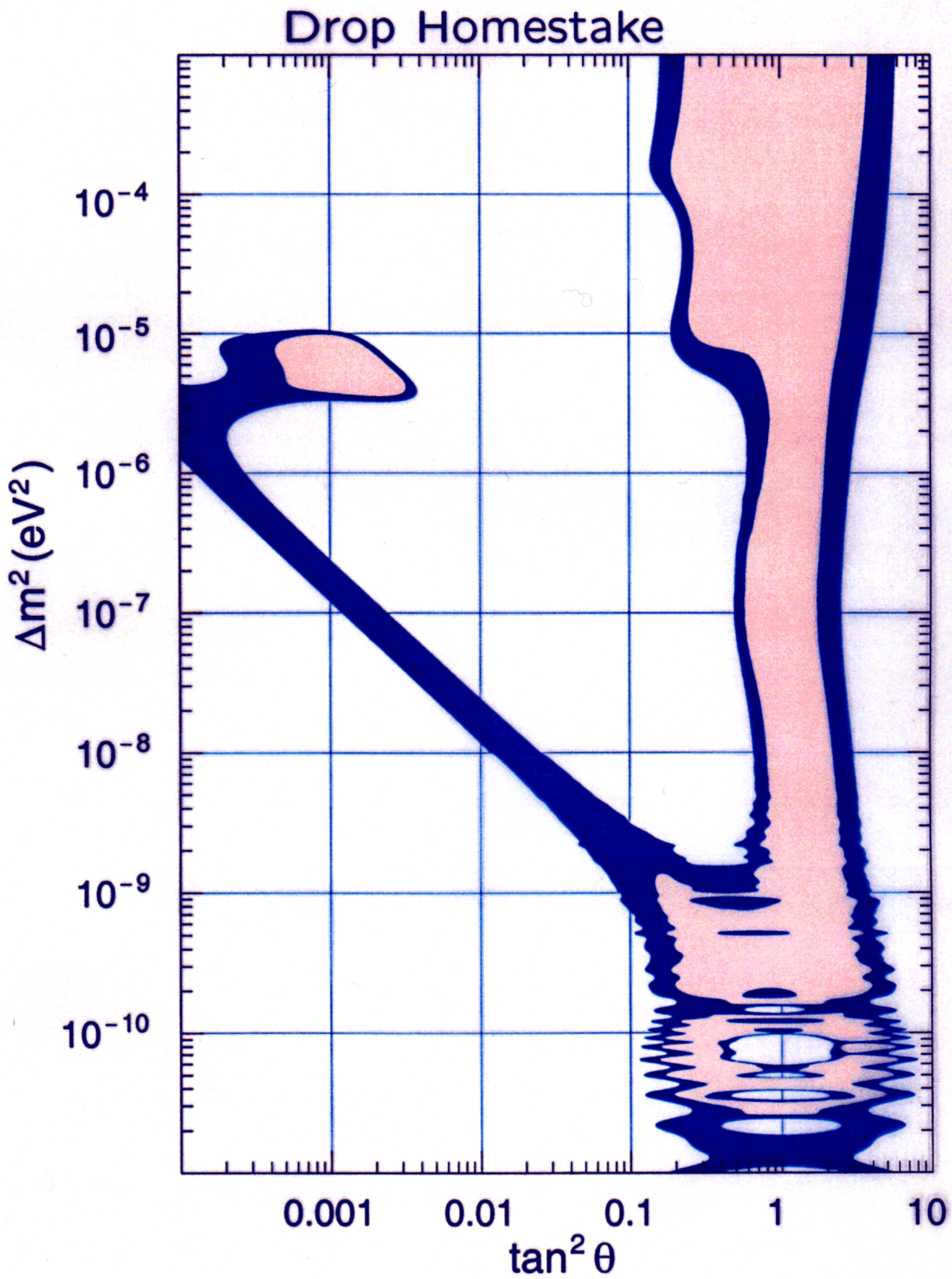
Allowed by Ga + SK + ChI rates at 3σCL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Δm^2 (eV^2)
C)
0.1
1
10
$\tan^2\theta$

Global view of the parameter space

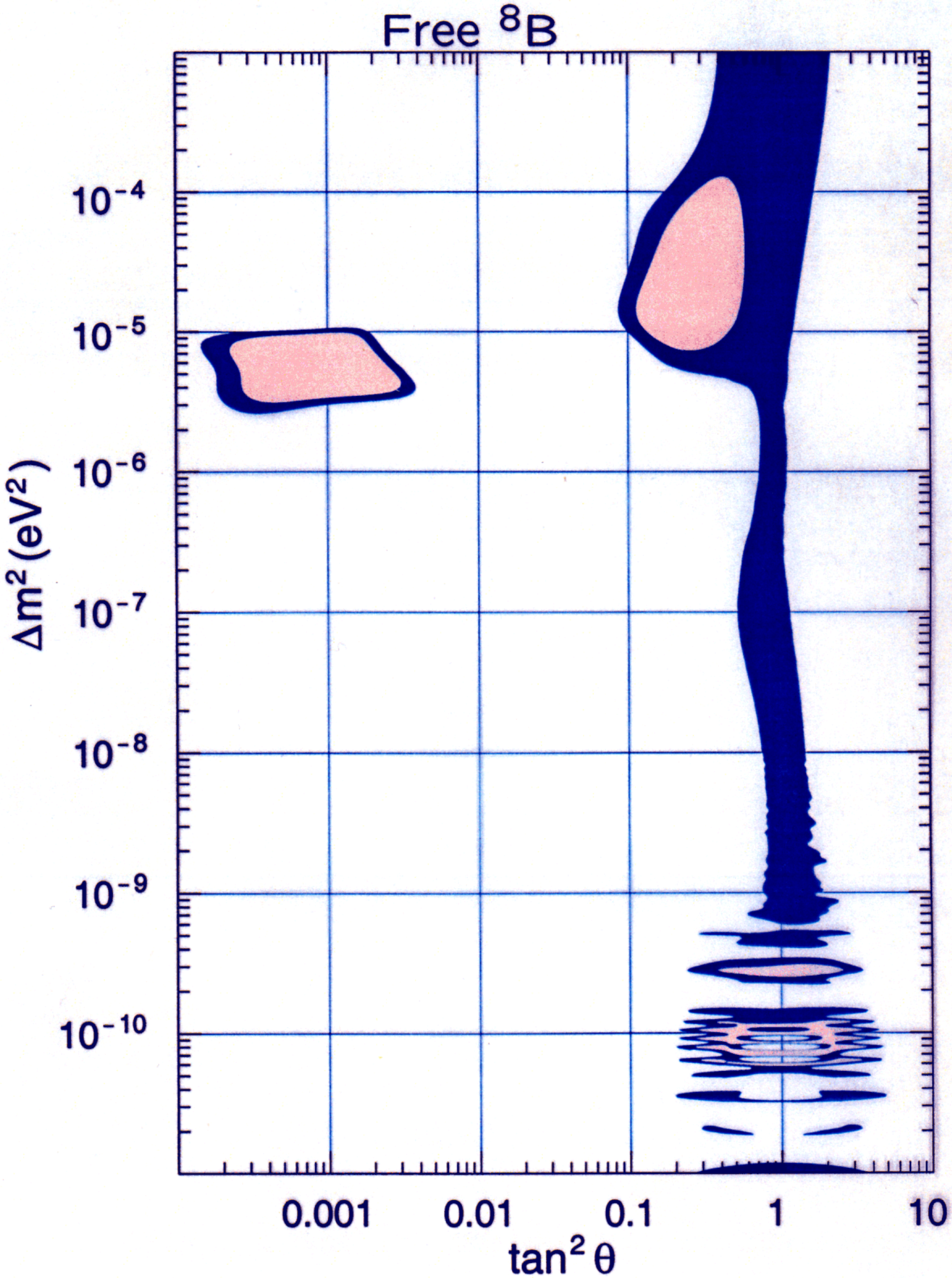


Excluded by day–night asymmetry at Super-Kamiokande





Drop Homestake
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$\Delta m^2 (eV^2)$
0.001
0.01
0.1
1
10
$\tan^2\theta$


Free ^{8}B
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$\Delta m^2 (eV^2)$
0.001
0.01
0.1
1
10
$\tan^2\theta$

Conclusions

- In the presence of matter effects the full physical range of mixing angle is $0 \leq \theta \leq \pi/2$. $\sin^2 2\theta$ is not a very good choice of parameter, $\tan^2\theta$ is better.

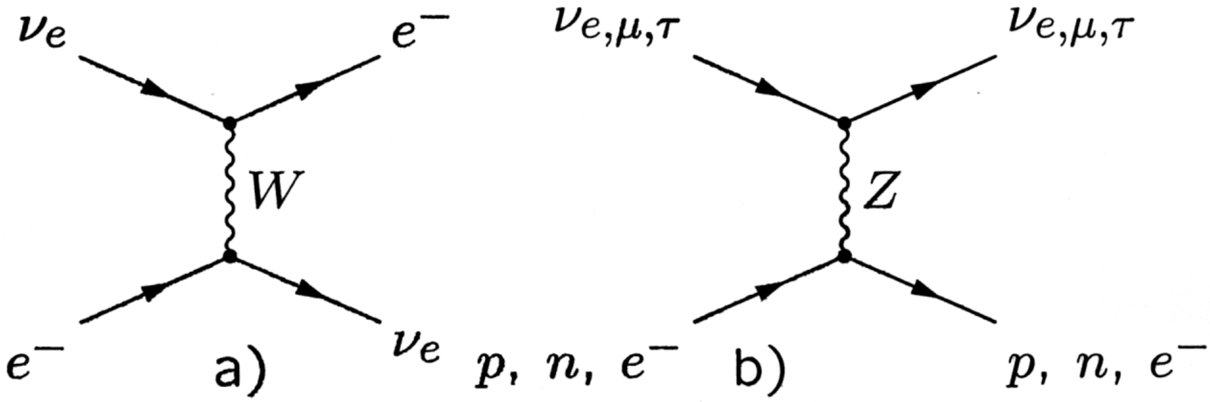
- MSW solutions are not necessarily confined to $\theta < \pi/4$.

- Matter effects for $\Delta m^2 \sim 10^{-10} - 10^{-9}$ eV^2 are non-negligible, especially for the low–energy *pp* neutrinos.

- It is useful to study the entire region $\Delta m^2 \sim 10^{-11} - 10^{-3}$ eV^2 at once, without separating MSW and vacuum regions.

- Experiments are urged to present their results on the both sides of the parameter space.

Matter effects



- Neutrinos interact with matter through the charged and neutral current interactions.

- Neutrino interaction cross section with matter is very small, $\sigma \sim G_F^2 s/\pi = G_F^2 2E_\nu m_e/\pi$. As a result, they are hard to detect.

- However, there is another effect. Neutrino interactions with matter lead to the *index of refraction* of the media.

Index of refraction is a *phase phenomenon*, related to the forward scattering *amplitude* for a process. It modifies the neutrino masses squared as follows:

$$\begin{aligned} m^2_{\nu_e} &\longrightarrow m^2_{\nu_e} + \sqrt{2} G_F E_\nu (N_e(x) - N_n(x)/2) \\ m^2_{\nu_{\mu,\tau}} &\longrightarrow m^2_{\nu_{\mu,\tau}} - \sqrt{2} G_F E_\nu N_n(x)/2 \end{aligned}$$

The charged current diagram exists only for ν_e.

Notice that the effect is proportional to G_F, not G_F^2.

If there is no mixing between neutrinos of different generations, the effect still would be unobservable. However, in the presence of mixing the matter terms can have a dramatic effect on neutrino oscillations by modifying the mixing angle and Δm^2 in matter.